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Is a triangle 180 degrees? Using relevant material to explore elementary school students' reasoning ability

Jihe Chen [a,1,*], Ying Zhou [a,2*], Tommy Tanu Wijaya [a,3], Maximus Tamur [b,4]

[a] Department Mathematics and Statistics, Guangxi Normal University, China
[b] Indonesia Santu Paulus Ruteng Indonesian Catholic University, Ruteng, Indonesia
[1] 1113952796@qq.com; [2] zhouying66@mailbox.gxnu.edu.cn; [3] tanuwijayat@gmail.com; [4] maximustamur@unikastpaulus.ac.id

* corresponding author

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ABSTRACT

Students' proof construction is one of the important abilities in learning mathematical knowledge. To understand the development of Students' proof construction of the fourth-grade students in Guilin, 15 fourth-grade students were selected to conduct an exploration activity about the sum of the inner angles of a triangle is equal to 180°. Through the observation and interview with students in the activities, we conclude that the reasonable ability of excellent students has coherence and integrity, the middle level students' proof construction is ordinary and still need to improve, and the Low Students' proof construction students are weak, and it is difficult to understand the research object. The teacher must have a creative and interesting method to teach students to improve Students' proof construction from the research process and results.

1. Introduction

Mathematical proof ability is very important and one of the higher order thinking skills [1]–[3]. Mathematics proof is referred to the researchers[4], based on existing knowledge experience, the emotion, individual intuition, under the influence of non-intelligence factors such as forehead in specific situation, the use of analogy, suppose, induction and another form of thinking, given the objective facts to conform to the experience, idealized proof, is a kind of uncertain proof. The sum of the inner angles of a triangle is a basic lesson geometry in primary school [5]–[7]. Proof construction is used to prove the sum of the angles of a triangle in elementary school [8]. Since elementary school students are not yet capable of rigorous proof, it is more appropriate to teach students mathematical ability[9], [10].

From primary school mathematics textbooks, we can clearly see that proof construction in the related learning content is involved in geometry [11], [12]. For example, determining and describe the length, area, and volume of an object through measurement and calculation rarely involves strict proof [13]. As far as the content of "sum of the inner angles of a triangle" is concerned, mathematical proof can be seen from various mathematics textbooks from China [14], [15]. All mathematics textbooks in China mostly contain the following contents: Provide material; Operation inquiry; Conclude. Among them, the operation exploration mainly includes measurement calculation, cutting, and folding (Fig 1-4, and Original (left) VS Translated (right)).

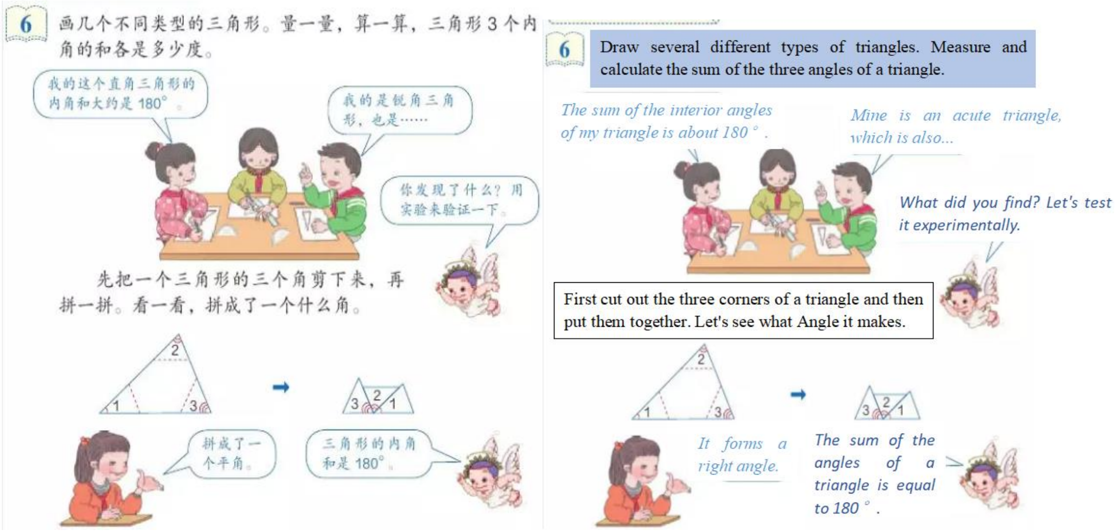


Fig. 1. PEP edition

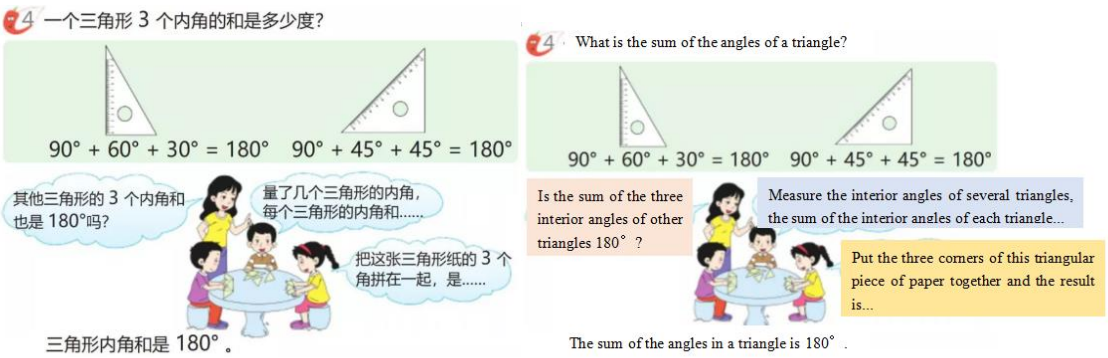


Fig. 2. SNUP edition

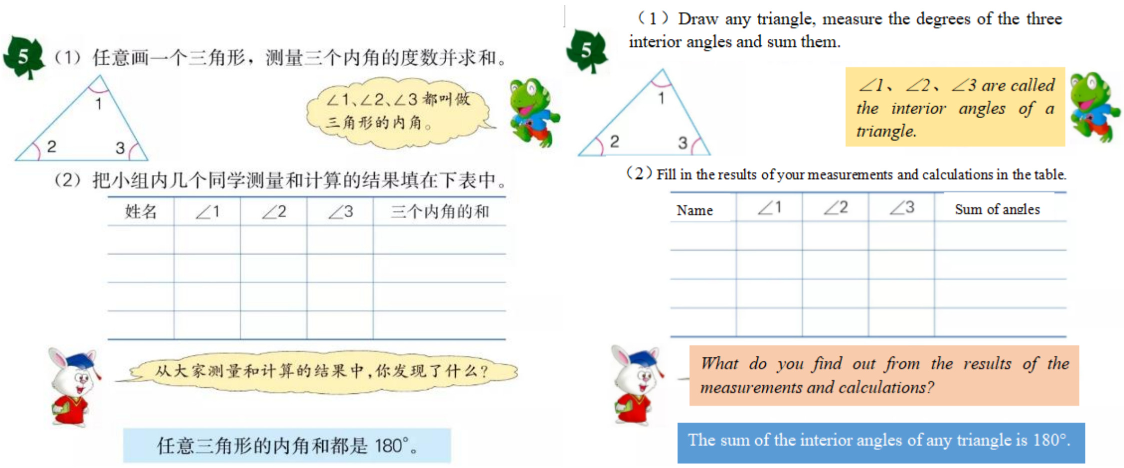


Fig. 3. HEP edition

Fig. 4. JEP edition

As shown in the above pictures, each version uses different proof methods, derivations and suggestive language to prove that the sum of the angles inside a triangle is equal to 180°. In order to facilitate the comparison of their differences and connections in the steps of proof, the designs of the four versions on the sum of the inner angles of a proof triangle equal to 180° are arranged in the form of Table 1.

Table 1. Comparison of different textbooks

Step	Textbook			
	PEP	*SNUP*	*HEP*	*JEP*
Create situation	Draw several different triangles, measure and calculate the sum of the three interior angles.	What is the sum of the angles of a triangle?	Draw any triangle, measure the degrees of the three angles and calculate their sum.	Do you know how many degrees the sum of the three internal angles of each square is?
Learning material	Each student prepares a triangle of a different shape	Two special right triangles	Arbitrary triangle	Two special right triangles
Operational verification	Measure, calculate, cut and splice	Calculate, measure, cut and splice	Measure, calculate,	Measure, calculate, cut and splice
Draw a conclusion	The sum of the internal angles of a triangle is 180°	The sum of the angles of a triangle is 180°	The sum of the angles of any triangle is 180°	The sum of the angles of a triangle is equal to 180°

It can be seen from Table 1 that the process of proving the sum of the inner angles equals 180° requires guessing, hands-on, and proof activities. According to many classroom studies and analysis of students' homework, we found that whether it is the intuitive operation or relatively abstract proof, students have some problems in the process of inquiry, mainly including the following 4 aspects (Fig 5). Students' visualization is gradually established in the activities. Recall the life experience [16], [17], observe the actual object, start to operate, imagine are the critical means to learn to understand the abstract geometry concept and to improve students' mathematical proof skills. From the point of mathematical content, sides and angles are important knowledge of triangles and must be mastered by students [18], [19]. Furthermore, this content has a very important value for the cultivation of students' proof construction [20]. The textbook allows students to get experience explaining that the sum of a triangle's inner angles is 180° through methods such as measurement angle. This activity increases the students' mathematical proof ability. This study focuses on developing teaching and learning activities that can improve students' mathematical proof abilities, helping students understand triangular angles.

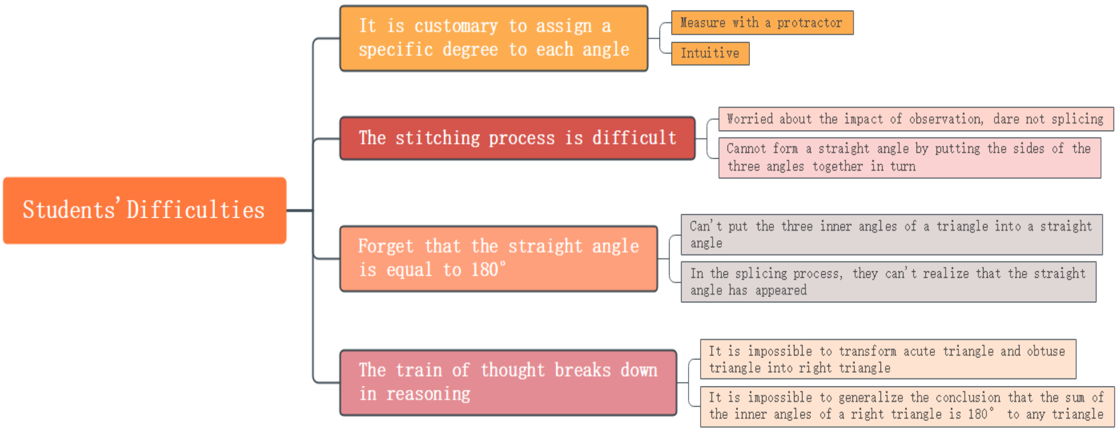


Fig. 5. Difficulties of students in the process of inquiry

Altunkaya et al [21] analyzed students' estimation achievements on the triangle material. This researcher analyzed 337 students from 3 elementary schools. The research shows a relationship between student learning outcomes and student learning motivation on triangle material. Altunkaya's research also shows that skill estimation on triangular material and geometry is important for students. There are some unanswered questions in altunkaya research, namely how to develop a curriculum to improve students' abilities on triangular material? What are the important factors in the third that students must master?. Abdullah [22] in 2020 researched student difficulties with the triangle and found that students had problems working on triangle problems. Abdullah further investigated the reasons for this difficulty using a qualitative approach. The research subjects were 55 grade 9 students. Furthermore Abdullah chose 5 students to find their reasons when working on the questions. Abdullah found that students had difficulties in overlapped triangles and Angle-Angle-type questions.

Haj Yahya [23] in 2021 conducts misconceptions research on triangular material. The results of the study indicated that many participants could not work on questions about the triangle concept. Based on the explanation and response from the interview section, students answered that they did not understand the theorems and definitions. Students have difficulty understanding the characteristics of triangles and this results in weak knowledge of the definition of geometry. Apart from the research above, there are still many short papers that discuss about triangle material [5], [19], [24], [25]. From several previous studies, researchers concluded that until 2021, misconceptions about triangles were still being found. The material of the triangle is still widely discussed and is important to research. There are still students with misconceptions in the triangle material. So the researcher with this research shows further research on how the students' reasoning abilities are when proving the angle in the triangle.

The significance of this study lies in using a variety of materials to understand the development of the useful reasoning ability of the fourth-grade students, and to evaluate the thinking level of the students through the process of hands-on operation. Understanding these can help teachers choose appropriate teaching methods pertinently in classroom teaching, so that students can accept some understandable proof process to learn mathematics knowledge. Compared with previous research, this paper's innovation lies in that it does not study the development level of reasonable reasoning ability of students of the fourth class by focusing on the inner Angle of triangle and this knowledge point at present.The experimental materials selected in this study cover various reasonable reasoning proof processes, including both the proof methods of general textbooks and more difficult reasonable reasoning methods. These proof methods can be combined to identify students' proximal development zone and their learning acceptance ability more effectively. This study mainly revolves around the following three questions; (1) What are the students' characteristics of different levels present when proofing the triangle's inner angle?; (2) How students think when proving that the angle in the triangle is equal to 180°?. (3) Can students transfer the inquiry methods obtained in a certain type of material to other related materials?

2. Method

Educational experts believe that the process of mathematical proving is an effective way to improve students' reasoning ability [1]. But every textbook proves that the triangle is 180°, each process has its own advantages and disadvantages. To fully understand students' reasoning ability, these materials should be put together (such as materials 1, 2 and 4), and designed into materials with strong abstraction and high reasoning ability (such as materials 2 and 5), to explore students' understanding and acceptance deeply. What materials can help students develop reasonable reasoning ability is also the concern of in-service teachers. Therefore, in-service teachers divide students into groups A, B and C (High, Medium, and Low) according to their daily performance and examination results, representing three types of students with strong reasoning ability, average reasoning ability and poor reasoning ability respectively, and invite them to cooperate with our research.

To ensure that students can help us to complete this research, this paper designs the research process and student interviews with the help of two education experts and one in-service teacher. Education experts are familiar with child psychology and behavioral knowledge and have a deep understanding of students' cognitive ability, reasoning ability and psychological activities. The in-service teachers have more than 10 years of teaching experience and are good at summarizing students' learning situations, learning style and learning ability. Their playbook expertise and teaching experience helped to ensure the rationality and execution of this study. This research adopts the method of experimental investigation and personal interview [26], [27]. This research was conducted in an elementary school in the Guangxi province of China. First, review the knowledge about angles and measuring angles with 15 fourth-grade students and explain the rules for using materials to prove that the angle in a triangle is equal to 180°. Then conduct experiments on each student individually, let the students explore and prove that the triangle angle is equal to 180°. At the end of the lesson, the researcher chose 3 students from high, moderate, and low levels to be interviewed. More detail about the research plan can be seen in Fig 6.

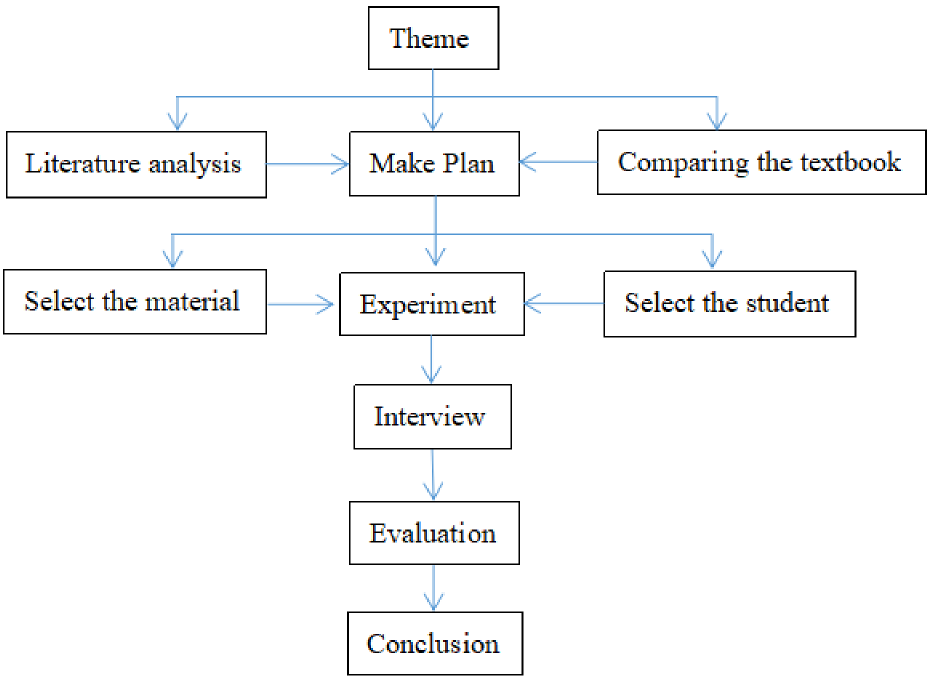


Fig. 6. Research framework

The exploration of reasoning ability of pupils. However, up to now, no one has designed to use five materials to prove the sum of the inner angles of a triangle to explore students' reasonable reasoning ability, so the design and research methods of this study are relatively novel.

2.1. Initial observation

Before the experiment, to enable all the students to use the tools normally and deduct the sum of the inner angles of a triangle, we reviewed the relevant knowledge for all the students; (1) How to measure the Angle; (2) The symbols of right angles; (3) Introduce the difference between 0° and 180°; (4) The value of the Angle can be added and subtracted.

2.2. Material preparation

students did 5 experiments with different learning media. The researcher provides experimental tools for students to prove angles in a triangle equals 180° degrees (see Fig 7). Each student is given a pencil, an eraser, and a clean draft paper. When a certain group's materials are not enough, students can add the group's materials. But each group of materials cannot be reused. For example, materials from the first group are not allowed to be used in the second group. Besides, each student conducts the investigation separately and does not limit each group of materials to explore. The actual completion time shall prevail. However, after the student has ended a certain exploration group, students cannot return to continue the exploration.

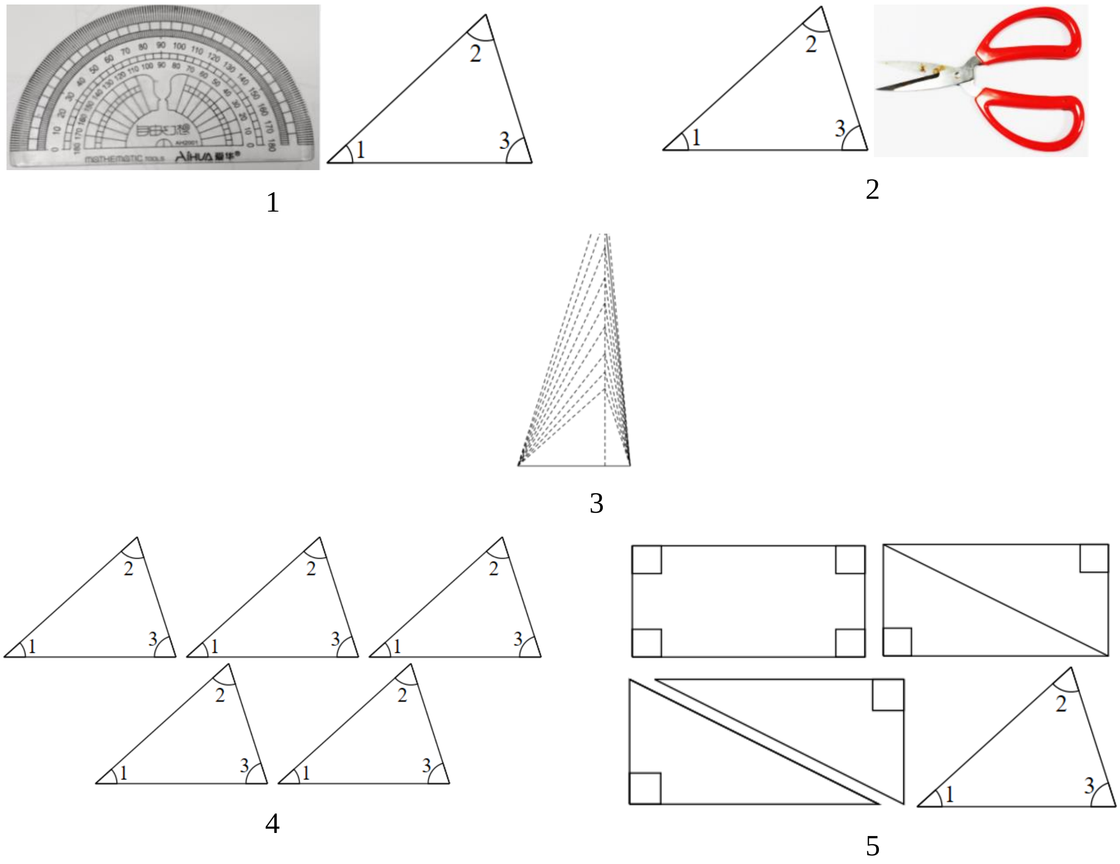


Fig. 7. Experimental tools for students to prove angles in a triangle equals 180° degrees

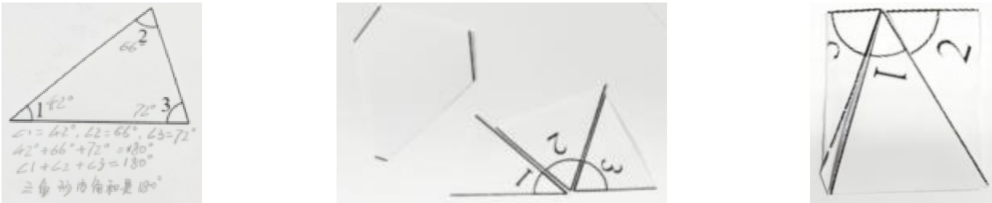
2.3. Experiment procedure

Before the experiment starts, the teacher asks the students to give their conjectures about the sum of the triangle's inner angles. Afterward, the teacher presented five sets of materials. Students were asked to observe the materials and ask questions about using the materials, and the researchers explained them appropriately, such as (1) Material 1: This is a triangle drawn on paper, and there is also a protractor; (2) Material 2: This is a triangle cut from paper. All students can handle it whatever you want; (3) Material 3: Imagine this is a rubber band tied to a nail plate. One side of the triangle does not move, and one vertex can move up or down (4) Material 4: This is a set of identical triangles (5) Material 5: The first three figures refer to a rectangle that has been divided into two right-angled triangles. Students can guess the sum of a triangle's inner angles based on the rectangle's change.

3. Results and Discussion

3.1. Manipulate materials and show the thought process

To understand students' cognition of the sum of the inner angles of triangles and test whether students can find the connection between different materials, the researchers asked them to choose five sets of materials to explore according to their wishes. The process of exploring each group of materials is recorded as follows Fig 8. Material No. 1 (calculation): This is no challenge for the 5 students in Group A, and the answer is quickly obtained. For the 5 students in Group B, except for B1 and B4 whose angles were not very accurate initially, the other 3 students quickly obtained the sum of the triangle's inner angles after simply observing the materials. For the students in group C1 and other students in group C, their expressions are slightly trivial, and they describe the specific measurement process and results.



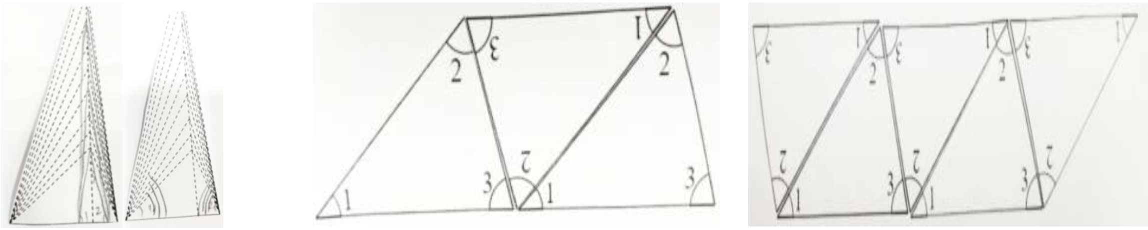
The calculation process of C1 Cutting and splicing Folding

Fig. 8. The calculation process of c1, cutting and splicing, and folding

No. 2 material (tear and put together): In front of No. 2 material, three students A1, A2, and A4 quickly reduced the three corners and completed the stitching. After thinking about A3 and A5, they adopted a folding method to fold the three corners' vertices into a flat angle. They can quickly switch between a straight angle and 180° and complete intuitive proofs in different ways. Although the three intermediate ability students were encouraged by the researchers to know that they could perform various operations such as folding, cutting, and tearing triangular pieces of paper, they could not find the direction in the specific operations. B1 first cut the triangle into a trapezoid and a triangle, and then cut the trapezoid into two triangles, trying to group together, but did not get the result; B2, B4 tried to find the multiple relationships of these angles by folding, and then changed ∠1. ∠2 is put together, but ∠3 cannot be put together; B3 and B5 also tried to put the three corners together by folding but found that the sides were not aligned and were at a loss. In the end, 5 students used the method of tearing, folding, or cutting to create a flat angle under multiple prompts.

When students with academic difficulties deal with No. 2 materials, they are even more purposeless. C2 and C4 traced one of the corners on the paper and made it a right angle, and then the train of thought was interrupted. Students C1, C3, and C5 with learning difficulties tried to fold the three corners of the triangle together with the researchers' encouragement and prompts, but they failed. In the end, only C1 and C4 formed a flat angle through tearing and splicing under the prompt of the researcher. Material No. 3 (Imagination): In addition to A3, the other four students in group A can imagine and describe according to the changes in the graph that when a vertex moves upward, the upper angle will become smaller and smaller, infinitely close to 0°, below The two angles will get bigger and bigger, infinitely close to 90°; when this vertex moves down, the upper angle gets bigger and bigger, infinitely close to 180° (or a straight angle), and the other two angles get smaller and smaller, infinitely close 0° (see Fig 9).

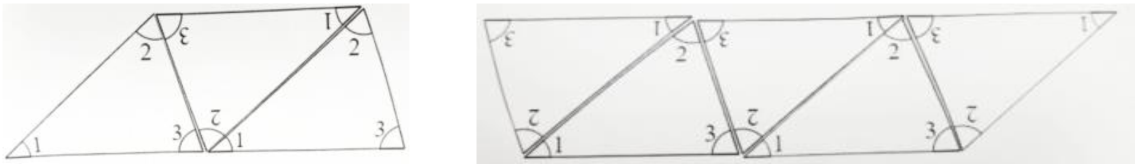
Under the guidance of the researchers, B2 and B5 of the B group students can imagine the three corners' changes, and the other three directly gave up the exploration. C5: I don't want to continue exploring. I don't want to continue this experiment; this experiment is difficult to understand why the vertices of a triangle can change indefinitely. I believe that such an approach is unfounded. Following the statement of C5, the students had difficulty understanding what it meant to explore this. Among the students in Group C, C1 and C5 indicate that there are too many angles, and it is difficult to measure the size of the highest vertex. No. 4 material (composition group): A1, A2, and A5 in Group A quickly associate with the relationship between No. 4 material and No. 2 material. Three triangles are selected, and ∠1, ∠2, ∠3 is used respectively. A3 tries to combine two congruent triangles into a parallelogram and uses the sum of the parallelogram's inner angles to be 360° to prove that the sum of the inner angles of the triangle is 180°.



Imagination of group A's Splicing process of A3 Splicing process of A4

Fig. 9. Imagination of group a's, splicing process of a3, and splicing process of a4

The researcher explained that "the sum of the parallelogram's inner angles is 360° ,it hasn't been proved yet, so it's better not to use this method. After that, he adopted the same method as the three students above (See Fig 10). Student A4 connected 5 triangles (Fig 10). He said that I now got three sets of triangle internal angle sum graphics. They are all straight angles, which means that the triangle internal angle sum must be 180°.



A3 A4

Fig. 10. Splicing process of A3 and A4

In group B, except that B1 and B4 can complete the proof process alone, B2, B3, and B5 all try to "forcibly" piece the same angle in each triangle into a circumferential angle, and then divide 360° by the number of angles to find the number of degrees out of this angle.

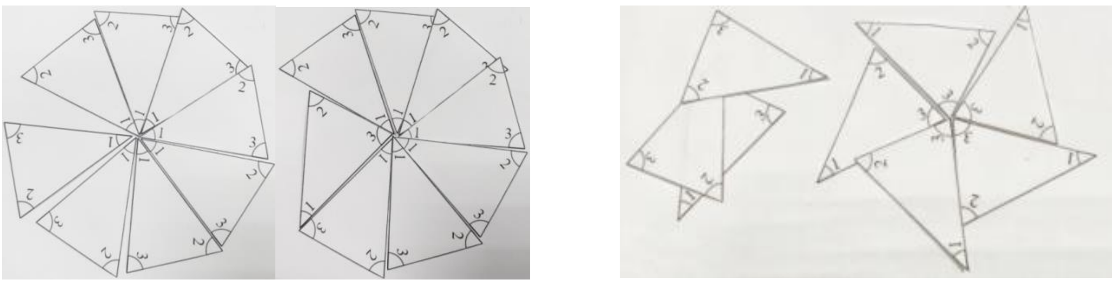


Fig. 11. Thinking process of group B

They did not realize that this did not fit precisely into the corner. It can be seen in Fig 11 that they are very obsessed with specific numbers, and computational thinking has played a considerable obstacle. In the end, the researcher prompts:

> *" Now, you tore off the three corners of material No. 2 (or by folding) and put them together because you only have one triangle. Now, material No. 4 has many triangles. You need to tear it. Is it?"*

After being prompted, they used three different angles to create a straight angle and realized the equivalent relationship between a straight angle and 180°.

The exploration process of group C students is complicated. Their thinking is often chaotic. Take students C1 (Fig 12) and C2 with learning difficulties as examples:

C1: I put ∠2 together because 3 ∠2 are equal to a right angle, so 180°÷3=60°, 180°+60°×2=300°.

Researcher: The problem is the three corners of the triangle, such as ∠1, ∠2, and ∠3 combined.

C1: Put together ∠2 and then ∠3 together to form a circle (circumferential angle). It takes several ∠1, ∠2 and ∠3, then I can add it up. Researcher: But it's not exactly 360° when put together.

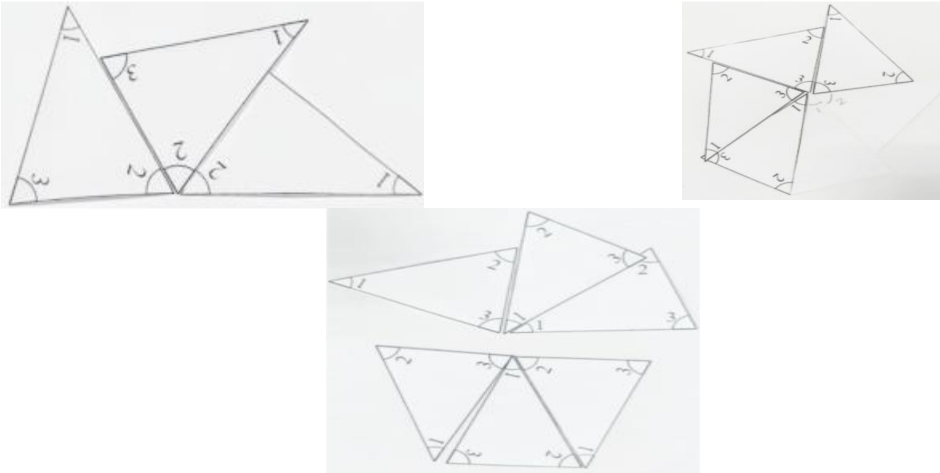
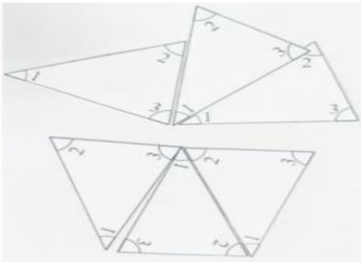
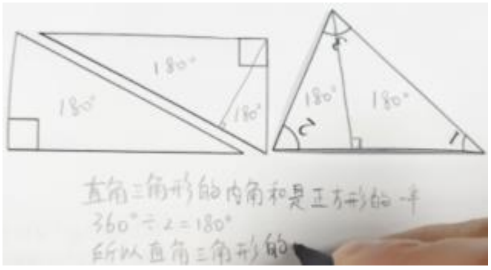


Fig. 12. The thinking process of student C1

C2 first tried to put together a group but didn't have a clue (Fig 13). Then he put ∠1, ∠2, and ∠3 together and immediately denied his idea. Soon he spelled out ∠1, ∠2, ∠3 again but still didn't realize that the problem had been solved and continued to meditate.



The thinking process of student C2



The thinking process of student A1

Fig. 13. The thinking process of students

The conditions of C3, C4, and C5 are like them, and their common feature is to operate for operation without a clear purpose. In the end, only C1 and C2 can be completed under the prompt. Material No. 5 (proof): The five students in Group A explored material No. 5 smoothly. A1 (Fig 13) , in, can write out the reasoning process very quickly. Their ideas are more consistent. Firstly, explain that the sum of the four corners of the rectangle is 360°, because 90°×4=360°; then explain that the right-angled triangle is 180°, because the rectangle is divided into two identical triangles, so 360°÷ 2=180°. When investigating the internal angle sum of an acute triangle, A3 and A5 need to be prompted by the researcher.

Researcher: "Can you find a right triangle form an acute triangle? Then, it can be divided into two right triangles by drawing the height of the acute triangle, and Explanation."

Researcher: "Because the acute triangle was divided into two right triangles, but the two right angles disappeared after the right triangle was assembled into a large acute triangle, so 180°+180°-90°×2=180°."

When students B1 and B4 in Group B face the No. 5 material, they try to estimate the degrees of the three acute angles first and then add them together. Under the interviewer's prompt, B1 and B4 finally divided the acute triangle into two right triangles along with the height, and then drew the conclusion. B3 and B5 can only conclude that the sum of the inner angles of a right-angled triangle is 180°, and it loses the motivation to explore further the sum of the inner angles of the acute-angled triangle. B2 expressed that he did not want to try to use material No. 5 for exploration. C1 and C4 can understand the sum of the right-angled triangles' inner angles in the figure as 180° according to the diagram. However, they cannot draw the analogy to other right-angled triangles, so they did not further study the sum of acute-angled triangles. Only C1 divided the acute triangle into two right triangles with the teacher's help, Then, C2, C3, and C5 gave up directly. The results of the students' responses are shown in Table 2. The results of 5 students' investigations using 5 kinds of materials are counted.

Table 2. Experiment completion

Kind	Group								
	A			*B*			*C*		
	I	P	N	I	P	N	I	P	N
1	5	0	0	5	0	0	5	0	0
2	5	0	0	0	5	0	0	2	3
3	4	1	0	0	2	3	0	0	5
4	3	2	0	2	3	0	0	2	3
5	3	2	0	2	0	3	0	1	4

[a] *I = finish independently, without prompting
[b] P = completion after prompt
[c] N = not finished.

It can be seen from Table 2 that the method of measuring and summing with No. 1 material can be easily understood and operated by students of three levels. The completion of No. 2 and No. 4 groups is also relatively high. Students in group A can actively discover the connection between the two, while students in group B and group C regard them as two independent objects. However, the connection between them can also be found with the teacher's help. Material No. 3 is more difficult for students in Group B and Group C. After proper explanations and prompts, they still have difficulty imagining the changing process of the triangle's three angles. The No. 5 material can form a clearer idea based on the diagrams and appropriate prompts for group A and some students of group B. However, it is necessary for the other students of group B and C to understand the (arbitrary) right triangle. It is difficult for the sum of the internal angles to be 180°. It is even more difficult to further infer that the sum of any triangle's internal angles is 180°.

3.2. Personal interview

To understand students' psychological changes and logical proof thinking during the experiment, under the guidance of education experts, the following six questions were designed from the knowledge level of understanding, cognition, and transfer. Following the hierarchical research method, A1, B1, C1 from the three thinking levels was selected as the interview objects. The interview results are shown in Table 3. Through the interviews with the students of A1, B1, and C1, it can be found that the thinking of the high ability students is quicker, coherent, deep, capable of effective migration, rich in spatial imagination, and the information and materials selected when solving problems are very purposeful. The five groups of materials can be divided into three categories according to different ways of cognition. Facing these three types of materials, students' Cognitive methods have the following characteristics.

Table 3. Interview Record

Question	No	Answer
Can you determine the sum of the inner angles of any triangle?	A1	Of course, the answer is 180°.
	B1	Yes, their sum is 180°.
	C1	Yes, it is 180°.
Which group of materials can prove that the sum of a triangle's interior angles is 180°? Talk about your thoughts on each set of materials.	A1	All 5 sets of materials can be proved. However, the first group of materials may cause inaccurate results due to measurement. The second and fourth groups are very similar. They can show that the three inner angles of the triangle are flat. The third group is more abstract, but I am sure it is At 180°, the fifth group can only turn all triangles into right-angled triangles and then get the sum of the inner angles of the triangles, but I don't know if there is any other way.

Question	No	Answer
Which group of materials can prove that the sum of a triangle's interior angles is 180°? Talk about your thoughts on each set of materials.	B1	I think except for the third set of materials, other materials can be proved. The first group measured 180°. The second and fourth groups can turn the three angles into a flat angle, which means that the sum of the inner angles of the triangle is 180°. The fifth group means to turn each triangle into two A right-angled triangle and then explain that the sum of the triangle's internal angles is 180°.
	C1	I think it is the first group and the fourth group. They are relatively simple and can be seen clearly. The materials of the other groups are too complicated.
In class, what kind of proof method do you like to use?	A1	Both are fine, but I would rather know the principle of the proof method of the third group of materials.
	B1	I like to use splicing, which is the second or fourth group of materials.
	C1	I like the first and fourth sets of materials, which are simpler.
Is the sum of any obtuse triangle's internal angles greater than the sum of the internal angles of any acute triangle?	A1	Incorrect, because the sum of the interior angles of all triangles is 180°.
	B1	Incorrect, the sum of their internal angles is 180°.
	C1	You need to know the triangle's specific shape and then measure it with a protractor to determine it.
If one of the angles in a right triangle is 30°, can you tell me the degrees of the other two angles?	A1	90° and 60°.
	B1	Should be 90° and 60°.
	C1	Only know one angle. There is no way to find other angles.

3.3. Evaluation

The first type is merging, namely No. 1, No. 2, and No. 4. Whether it is measurement calculation, cutting and tearing, or grouping, you can combine the three inner corners of the triangle through specific operations. This kind of path is relatively easy to accept for all researchers, and it is also in line with their cognitive level. Among them, calculation greatly influences middle and lower students, they cannot be separated from the specific quantity, and they are always stuck in each part. , And can't look at things from a holistic perspective. The second category is proof is No. 5 material. The sum of a general triangle's inner angles is obtained from the sum of a relatively simple right-angled triangle through logical operations. For this activity, the high ability students' only they can transform ordinary triangles into right-angled triangles. Once the student realizes that the teacher can divide a general triangle into two right-angled triangles and the height, then the following logical operations are not a problem. Middle-level students can understand the relationship between the sum of the inner angles of a right-angled triangle and the corresponding sum of a rectangle's inner angles. However, there are specific difficulties in converting a general triangle into a right-angled triangle and performing subsequent logical operations. However, students with learning difficulties are struggling with this path.

The third category is imagination, such as material No. 3. Through the movement of the three corners of a triangle, it is found that the sum remains unchanged. The understanding of this material requires a strong spatial imagination. In the same way, the top students also have obvious advantages. The high ability students have their ideas on many issues. Even if the teacher does not teach them, they can observe and discover by themselves. The middle and lower students are obvious in this regard. More passive. In the process of exploring, the students can always focus on the target. Even if there are sometimes thought problems, the students can continue with a bit of hint. Middle school students are easy to forget the goal of their current behavior. When solving the same problem, the steps before and after cannot be effectively connected. They are easy to make judgments based on observation and intuition, and they are also easy to fall into a fixed mindset and are not open enough. Students with academic difficulties often fall into such a dilemma: they cannot understand what the current research problem is, they have no clear ideas, and they are easily lost in the process of research. Their expressions are also very isolated, fragmented, and lack of integrity.

Through the process of the experiment and the results of the interview, we sorted out the answers to three research questions. First, the fourth-grade students have different characteristics in proving the triangle angle. The students with strong ability to prove their thinking have good coherence and adaptability and are good at finding the general rules hidden between things from the related things.

They can make a preliminary transformation from concrete form to abstract thinking in the existing cognitive system. Middle-level students have general empathy reasoning abilities, and their thinking is characterized by integrity and stability. They are used to extract some experience from existing knowledge to supplement and explain empathy reasoning. Therefore, sometimes external help is needed to achieve the purpose of learning. But they have a quick adaptability to common proof materials and can realize the process of proof with the help of teachers. The students with learning difficulties have the most unsatisfactory reasoning ability. Their thinking is single and discontinuous, leading to their inability to understand the research object smoothly and cannot establish an effective correlation between materials and results. When the process of inquiry is beyond their cognitive range, they tend to show the psychology of retreat or abandonment.

Secondly, in the process of proof, the ideas of fourth-grade students are different. Students with strong ability to prove the unknown tend to use the existing knowledge to prove the unknown conclusion. They can clearly state the proof principle of various materials, so they are not satisfied with measuring and calculating angle. They want to find simple and practical methods. Because they have divergent thinking, their inner thoughts will be more active, grasp the characteristics of various materials, and then draw more abstract conclusions. The idea of middle-level students is to complete tasks according to the existing knowledge and experience step by step. Their reasoning ideas will be influenced by learning tasks and learning materials, and they cannot be bold in the association and integration of relevant materials. For students with difficulty in learning, they tend to study simply and directly. In the experiments of proof, they will also actively seek help, hoping to achieve certain goals with the help of teachers or others. This also shows that there is uncertainty in proving the sum of the inner angles of the triangle, and reliable reasons cannot explain it.

Finally, in converting these five kinds of proof materials, only the students with strong proof ability can successfully realize the process of different materials conversion. They use the existing information and their own abstract understanding ability to spread and explain the use of materials, and their thinking has exceeded the same teaching objectives. For students with general and poor ability of proof, their cognition is one-sided and limited. They cannot realize that different materials have the same effect on the same result. However, it is precisely because they know that their cognitive ability is limited. They will realize clearly that they cannot complete the process of proof and directly end the proof of a specific material. From this point of view, in the teaching process, we should choose the appropriate materials according to the students' acceptance level, and in the teaching process of a class, we should also adopt a various of teaching materials to build more platforms for the development of students [28]–[30].

4. Conclusion

The results show that students with different learning abilities have different levels of reasoning ability. Students with strong learning ability show good reasoning ability, students with medium reasoning ability are average, and students with poor learning ability are not ideal. This is related to their way of thinking and conversion ability, and the process of exploration will also be affected by the learning materials. Education experts point out that since the triangles in the textbook are static, special, and limited and do not represent all triangles well, dynamic animation effects can be designed to demonstrate the fact that the sum of the interior angles of different triangles is 180°. In this way, students with different reasoning ability can realize intuitively that the sum of the inner angles of a triangle is unchanged from the change process of the appearance of a triangle based on the general measurement of angles and calculation of the sum of the inner angles. On the one hand, static knowledge can be changed into dynamic knowledge to increase the interest of mathematics learning. On the other hand, it is beneficial to pass on the mathematical ideas combining special and general to students and lay a good foundation for students to engage in a more reasonable reasoning process. This research is based on the synthesis of various triangle interior angles and materials to study the reasonable reasoning ability of the fourth-grade students. For other materials or other conclusions, what is the situation of students' reasonable reasoning ability? Relative to the development of reasonable reasoning in fourth graders, what is the status of reasonable reasoning in other grades? Further research is needed.

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